Exercise 20

Find the area of a triangle bounded by the y axis, the line $f(x) = 12 - 4x$, and the line perpendicular to f that passes through the origin.

Solution

The equation of the y-axis is $x = 0$, the line $y = 12 - 4x$ is given, and the line perpendicular to $f(x)$ has the negative reciprocal slope, $1/4$. Use the point-slope formula and the fact that it passes through the origin $(0, 0)$ to get the equation of this line.

$$y - 0 = \frac{1}{4}(x - 0)$$

$$y = \frac{1}{4}x$$

Graph all three equations.
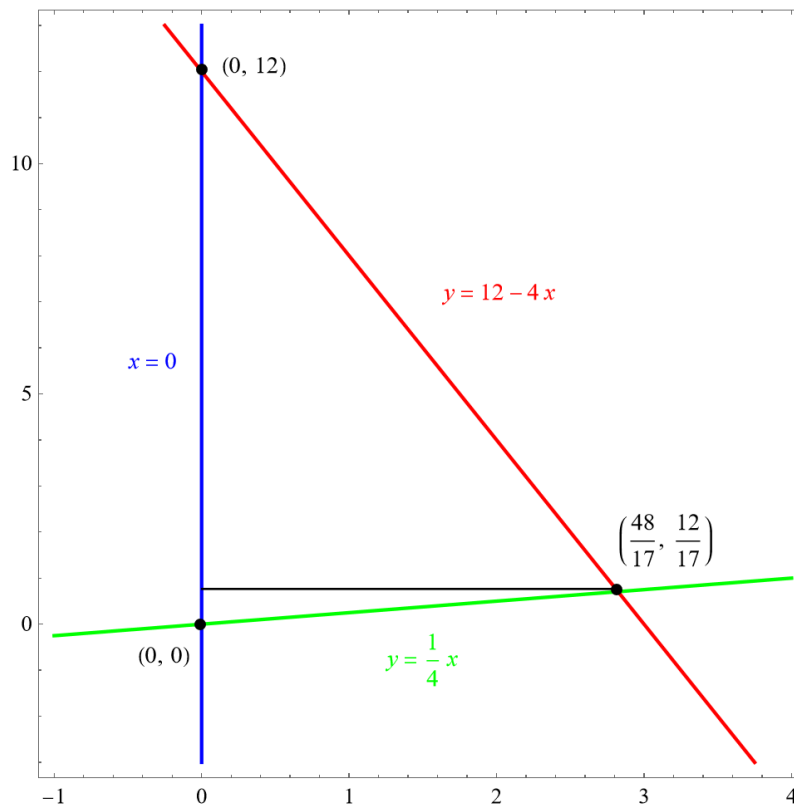
The area of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(12)\left(\frac{48}{17}\right) = \frac{288}{17} \approx 16.94.$$

The point of intersection on the right is found by setting the equations of the lines equal.

$$\frac{1}{4}x = 12 - 4x$$

$$\frac{17}{4}x = 12$$

$$17x = 48$$

$$x = \frac{48}{17}$$

To get the corresponding y-value, plug this into either equation.

$$y = \frac{1}{4}x = \frac{1}{4}\left(\frac{48}{17}\right) = \frac{12}{17}$$

Therefore, the point of intersection on the right is $\left(\frac{48}{17}, \frac{12}{17}\right)$.